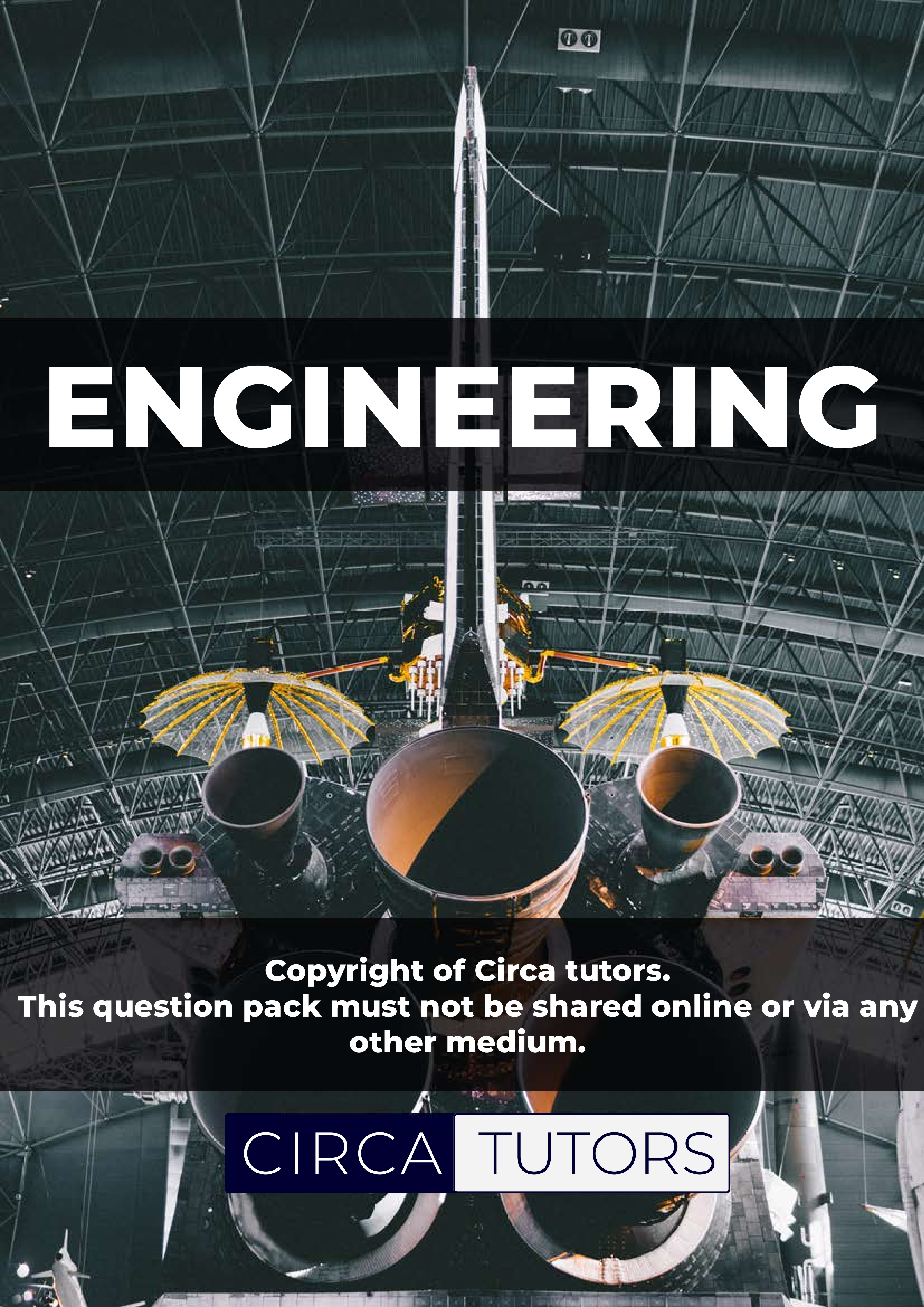


ENGINEERING

CIRCA TUTORS

Turbines and Power

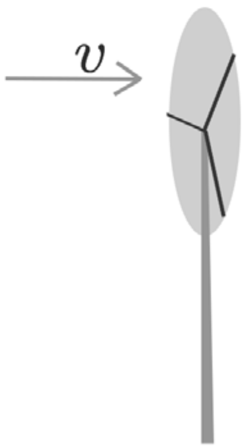
Required topics: Energy/ Power **Difficulty:** 3/5

Intro Questions [3 mins]

$P = Fv$ applies to an object travelling with a constant velocity,

If the body has a constant velocity, hence constant kinetic energy, then how is energy transferred to the object?

Main Question [8 mins]



a) [**4 mins**]

Shown in the diagram, is a wind turbine with air approaching at velocity v. The area that the turbine blades are able to cover, A, is shown in the grey circle. After the wind travels through the turbine blades its velocity decreases to 0. Assuming the wind turbine is 100% efficient, show that the power generated by the wind turbine is:

$$Power\ generated = \frac{1}{2}\rho A v^3$$

Where ρ is the density of the air.

b) [**1 min**]

Explain some problems that exist with this model

c) [**3 mins**]

Show algebraically ,that an increase in the wind velocity of 10% leads to an increase of approximately 30% in the amount of power generated

Extension Question [9 mins]

A rigorous analysis shows that a wind turbine that is 100% efficient will have the following equation for its output.

$$P = \frac{1}{4}\rho A v_1^3 \left(1 + \left(\frac{v_2}{v_1}\right) - \left(\frac{v_2}{v_1}\right)^2 - \left(\frac{v_2}{v_1}\right)^3\right)$$

Where v_1 is the speed of the incoming air and v_2 is the speed of the air after it has passed through the turbine.

a) Rewrite this equation in the form:

$$P = power\ incident\ on\ turbine \times f(\alpha)$$

$$(Where\ \alpha = \frac{v_2}{v_1})$$

b) Sketch a graph of $f(\alpha)$ for a suitable range of α

c) Discuss what the shape of the graph and the values obtained mean for the wind turbines maximum power output

Hints

Intro question

$P = Fv$ applies when the forward thrust is equal to the force of drag.

Main Question

a) Consider the change in the kinetic energy of the air per second
b) Is it reasonable to assume that the velocity of the air decreases to 0 after passing through the area covered by the wind turbine blades?
c) (Use the approximation that $(1 + x)^n \approx 1 + nx$ for small x)

Extension Question

a) An expression for the power incident on the turbine was determined in the main question part a)
b) Can α be negative? Consider what happens at $\alpha = 1$ and $\alpha = 0$
try finding the turnings points of α
c) Which value of α maximises the power output?

Solutions

Intro Question

The equation $P = Fv$ applies when the force of drag is equal to the thrust force. This occurs when a car is travelling at a constant speed. The power in this equation refers to the work done per second against the force of drag.

The kinetic energy of the body does not increase as all of the work done is against air resistance.

Main question

a)

Consider a cylindrical column of air that is incident on the turbine blades. The area of the column is A, hence the volume of air incident on the turbine blades per second is Av.

Multiplying by the density of the air ρ we obtain the mass unit time of air incident on the turbine blades. This is ρAv.

The change in kinetic energy per second of the column of air gives the power incident on the turbine. This is

$$P = \frac{1}{2} \times mass\ per\ second \times v^2$$

Substituting our expression for mass per second into the power, this gives

$$P = \frac{1}{2}\rho A v^3$$

We have assumed that the wind turbine is 100% efficient so this is equal to the power output of the turbine.

b)

One problem that exists with the model is that it is unrealistic to expect that the turbine absorbs all of the incident power from the wind. This is because there are only three turbine blades so some of the air will be able to pass through the gaps and around the blades.

We have also assumed that the turbine is 100% efficient which is impossible to achieve in practice as there will always be some resistance in its wires and some energy will be lost to sound and heat.

c)

This solution uses the binomial expansion given in the hint:

(Use the approximation that $(1+x)^n \approx 1 + nx$ for small x)

Performing this expansion in v. If x =0.1, this leads to approximately a 30% increase in the power incident on the turbine.

A strong candidate will interpret the physical significance of this. This shows that it is much more cost efficient to have a wind turbine in a windier area and that even small differences in the wind speed can drastically affect the power that can be generated by the turbine.

Extension Question

a)

Using the result in the main question a)

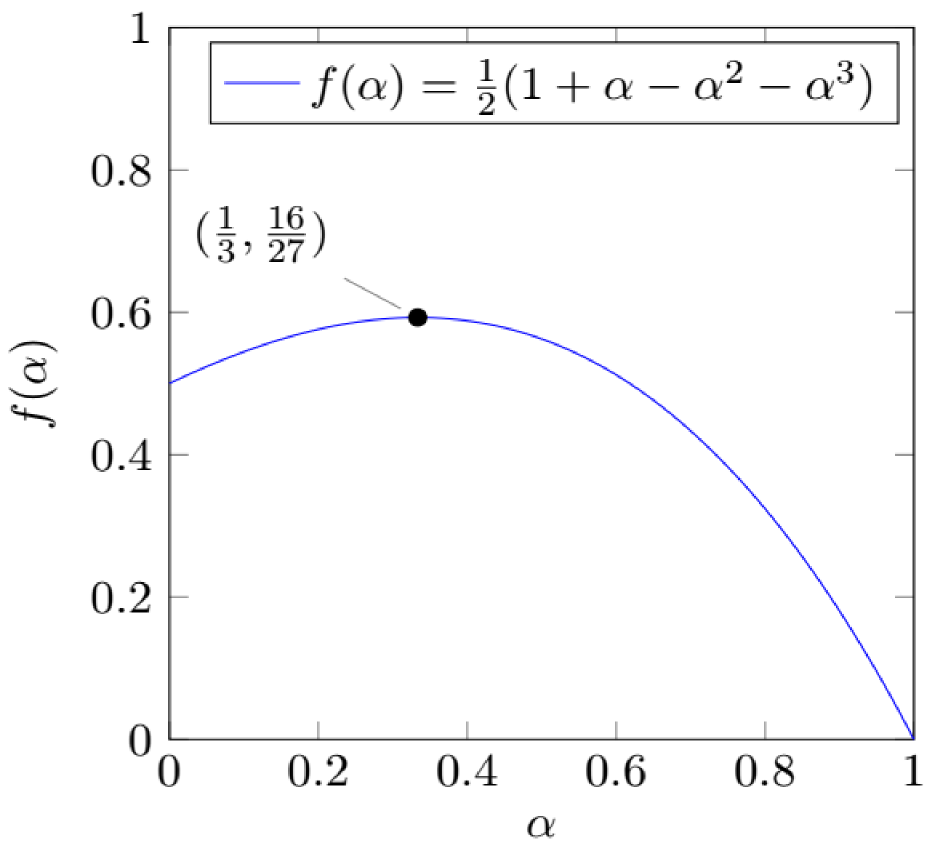
We find that:

$$P = \frac{1}{2}\rho A v_1^3 \times \frac{1}{2}\left(1 + \left(\frac{v_2}{v_1}\right) - \left(\frac{v_2}{v_1}\right)^2 - \left(\frac{v_2}{v_1}\right)^3\right)$$

Hence $f(\alpha) = \frac{1}{2}(1 + \alpha - \alpha^2 - \alpha^3)$

b)

Sketching this function:



We can differentiate to show that there is a maximum at $\alpha = \frac{1}{3}$, it is can be seen that this is a maximum by considering that this is a negative cubic graph with two turning points at $\alpha = -1$ and $\alpha = \frac{1}{3}$.

The range of values for α to consider are $0 \ < \alpha \ < 1$

α cannot be greater than 1 as the velocity of the air after travelling through the turbine cannot be greater than it was before it approached the turbine.

α being less than 0 would imply that the air has reversed its velocity which means that it has not passed through the turbine but rather bounced off one of the blades.

c)

The maximum power input for the turbine occurs when $\frac{v_2}{v_1} = \frac{1}{3}$

This is important to consider when designing the blades of wind turbines. Intuitively we would expect that $\frac{v_2}{v_1} = 0$ would extract the maximum amount of work from the wind, however this is not the case as shown by the equation in extension part a).

INTERVIEW SCORECARD

Candidate Name		Date	
Score/25		Question	

General Interview Notes

	Points to look for	Notes	Score /5
Explanation of ideas	How clear are the explanations, is the candidate explaining their thinking or are there long periods of silence? Is the candidate presenting their ideas in an organised manner both on paper and verbally?		
Problem solving ability	Has the candidate thought about the method that they are using or are they rushing into the solutions? Has the candidate tried to approach the problems using a few different methods?		
Response to new information	Does the candidate take a new approach to questions? Is the candidate ignoring hints or are they using them to inform their approach and solution		
Creativity	Does the candidate suggest a few good quality ideas on questions which require this?		
Accuracy	Is the candidate able to check their mistakes and spot their errors?		

Question Specific Notes (Fill in as appropriate)

Question number	Notes
Introductory question	
Main Question 1	
Main Question 2	
Extension Question	

Notes on the Questions for the Interviewer

Timings

The questions have been designed to stretch the highest ability candidates. It is worth noting that the timings on the questions are rough guides and that interviews will vary in length. Although it is unlikely that an academic interview will last less than 20 minutes, the interview could go on for up to 40 minutes.

The timings are designed to cater for all candidates and so when starting off it is likely that candidates will take much longer than the suggested time for the questions, especially with the harder questions.

Difficulty

The harder the question the more strict the timings are, only the most able candidates will be able to keep to the timings on the 4/5 and 5/5 questions so use the timings as a rough guide. The questions are designed so that a candidate should start off with the easiest questions and then work their way up to the harder questions.

Requirements

The candidate should be given a pen and some lined paper for the questions. They have been designed so that a calculator is not needed.

Mistakes and Feedback

Great care had been taken to ensure that the question packs are error free, however if you do spot any mistakes please email **circatutors@gmail.com**. If you have any other positive or negative feedback about the packs we would be really interested to hear from you, please email **circatutors@gmail.com.**